

percolation

robustness

cascading failure

SYSM 6302

CLASS 12

Percolation



- node (site) percolation - removing a fraction of nodes in a network
(and the adjacent edges)
 - edge (bond) percolation - removing a fraction of the edges in a network
- Removal can represent failure / destruction / death.
- Removal isn't always a bad thing: vaccination "removes" people from a contagion network

Occupation probability ϕ



$\phi = 1$: no nodes removed, "occupied" nodes are functional

$\phi = 0$: all nodes removed

As $\phi = 1 \rightarrow \phi = 0$, there is a percolation threshold at which point a giant component (giant cluster) dissolves.

Disease Spread

ABOVE :

Epidemic!

disease is confined
to small sections of
population

BELOW :

Internet

Most nodes can
communicate

Cannot reach
all nodes!

PERCOLATION
THRESHOLD



Percolation can be random (mimicing failures)

or can be strategic, e.g. by degree (mimicing attacks)

The configuration model captures most of the major percolation properties in an analytic framework.

↳ We will look at them empirically through Albert 2000

Practical Percolation



→ Instead make a fixed number of nodes (r) occupied

$$P_r = \binom{n}{r} \phi^r (1-\phi)^{n-r}$$

probability that r nodes are occupied
given occupation probability ϕ

$$S(\phi) = \sum_{r=0}^n P_r S_r = \sum_{r=0}^n \binom{n}{r} \phi^r (1-\phi)^{n-r} S_r$$

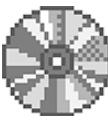
expected size of
largest component
as a function of ϕ

S_{r+1} is a minor update from
 $S_r \rightarrow$ it involves adding one
more occupied node

↑ cluster change is minimal & with
careful book keeping can be an easy
update

Cascading Failures

- A single failure can lead to successive failures →
 - ↳ often linked to capacities
 - ↳ fundamentally due to an underlying process



Power grids
Biochemical cascade
Finance (systemic risk)
traffic
...

